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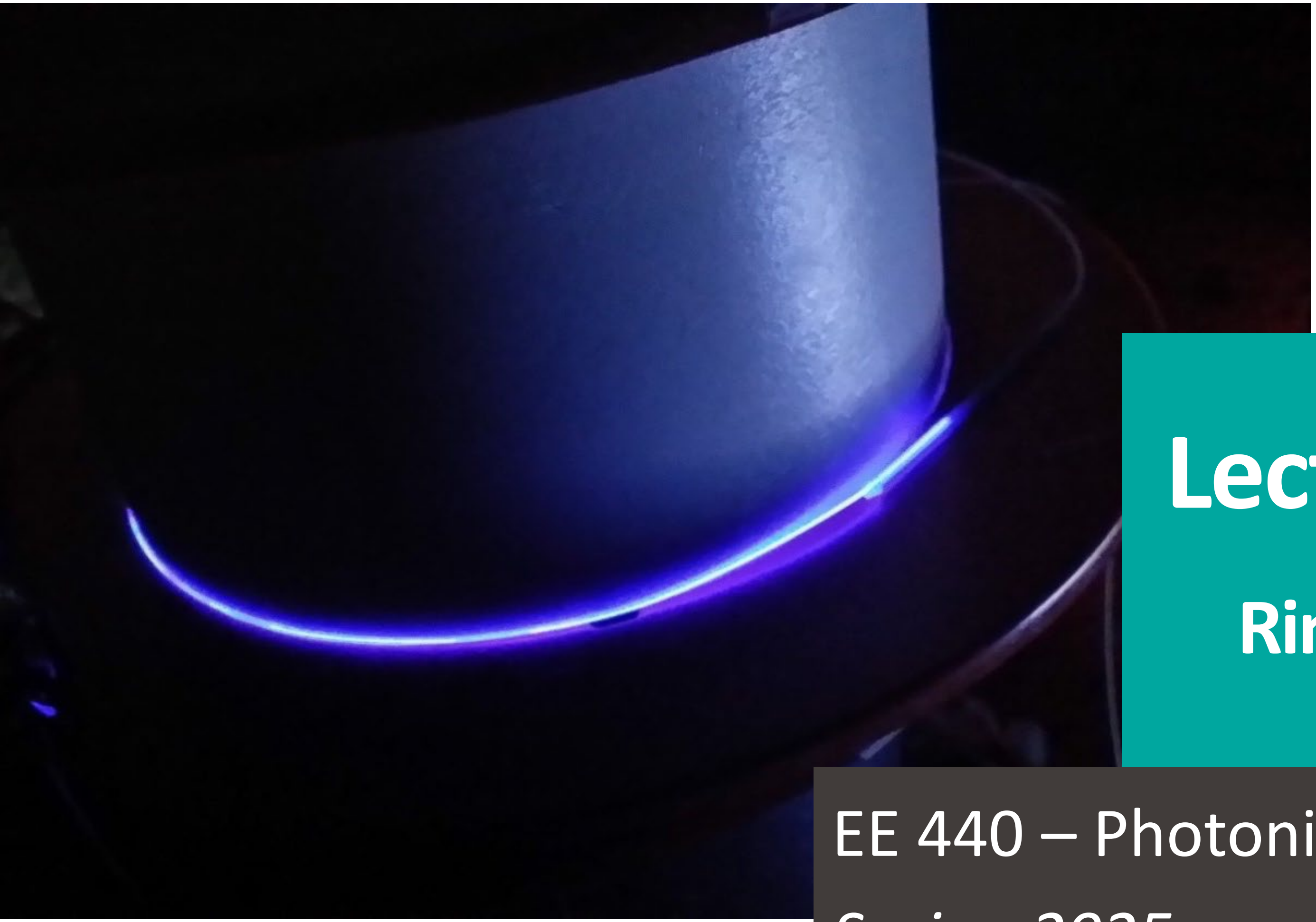

# Lecture 4

## Ring resonators

EE 440 – Photonic systems and technology

*Spring 2025*

Ecole
Polytechnique
Fédérale de
Lausanne

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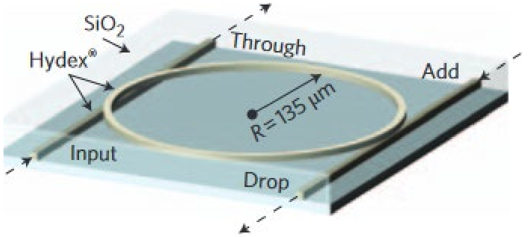
# Lecture 2 outline

Principle of microring resonator

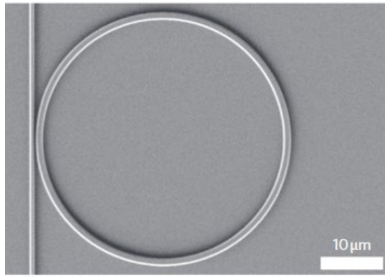
Power coupling formalism

Temporal coupled mode theory

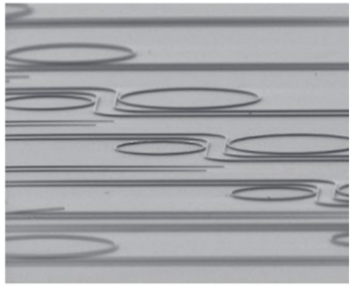
Microring resonator modulators



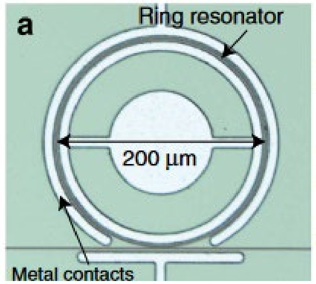

***Hydex***
Razzari *et al*. Nature Photonics 2009



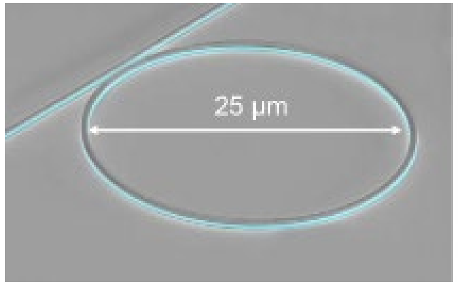

***SiN***
Levy *et al*. Nature Photonics 2010



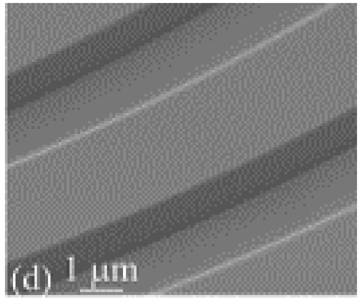
***Diamond***
Hausmann *et al*. Nature Photonics 2014



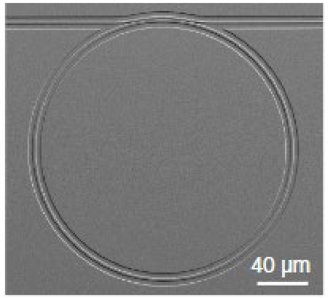

***Si***
Griffith *et al.* Nature Com 2015



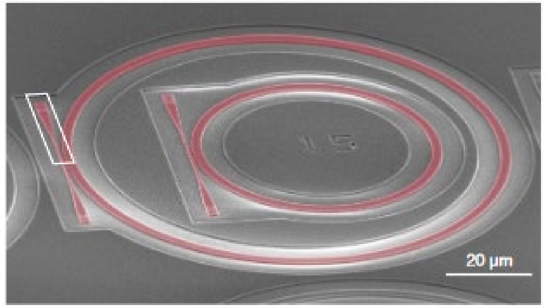

***AlGaAs***
Pu *et al*. Optica 2016



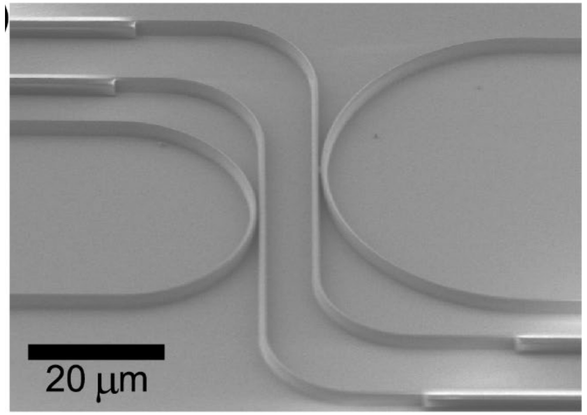

***AlN***
Liu *et al.* ACS 2018

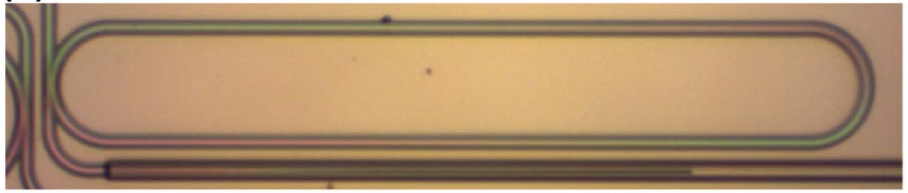


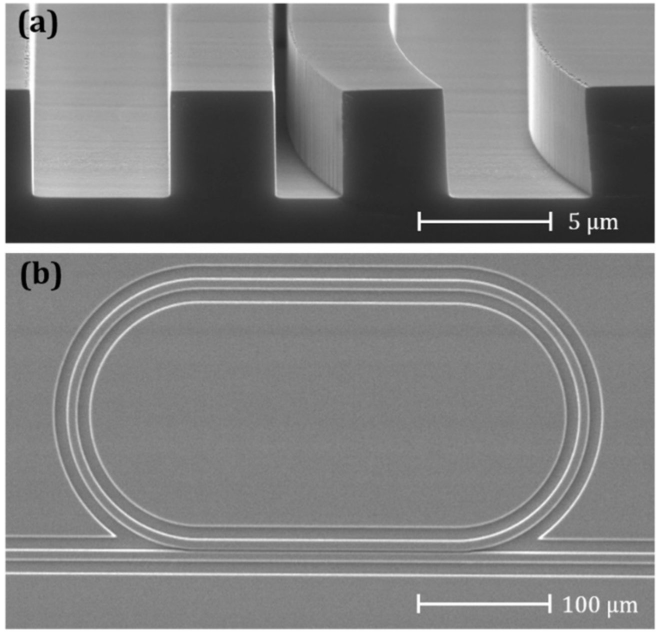

***$LiNbO_3$***
He *et al*. Optica 2019

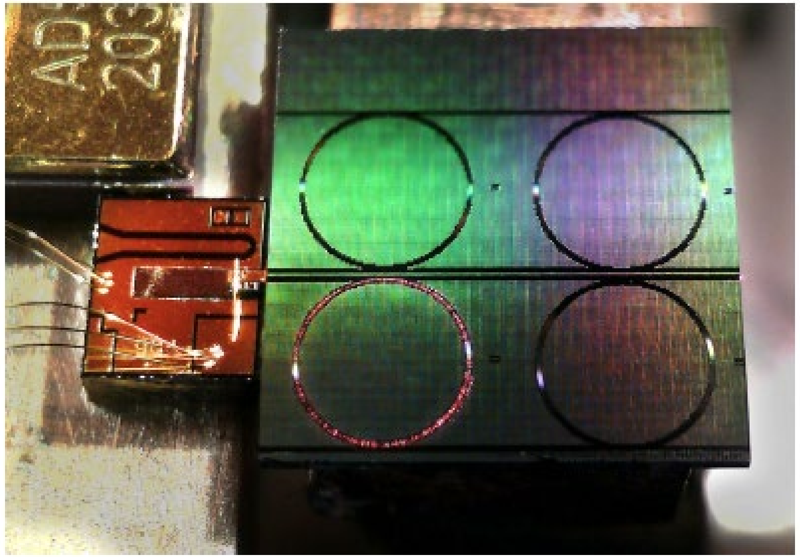



***SiC***
Guidry *et al*. Optica 2020


20 µm

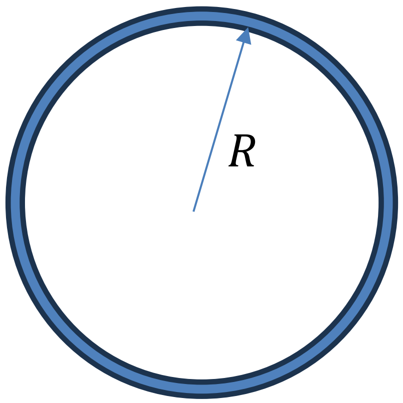



(a)
5 µm
(b)
100 µm



# Principle of microring resonators – power coupling formalism

# Ring waveguide




Assume there is light circulating inside the ring

Resonance condition: $\exp[-i\beta(2\pi R)] = 1$

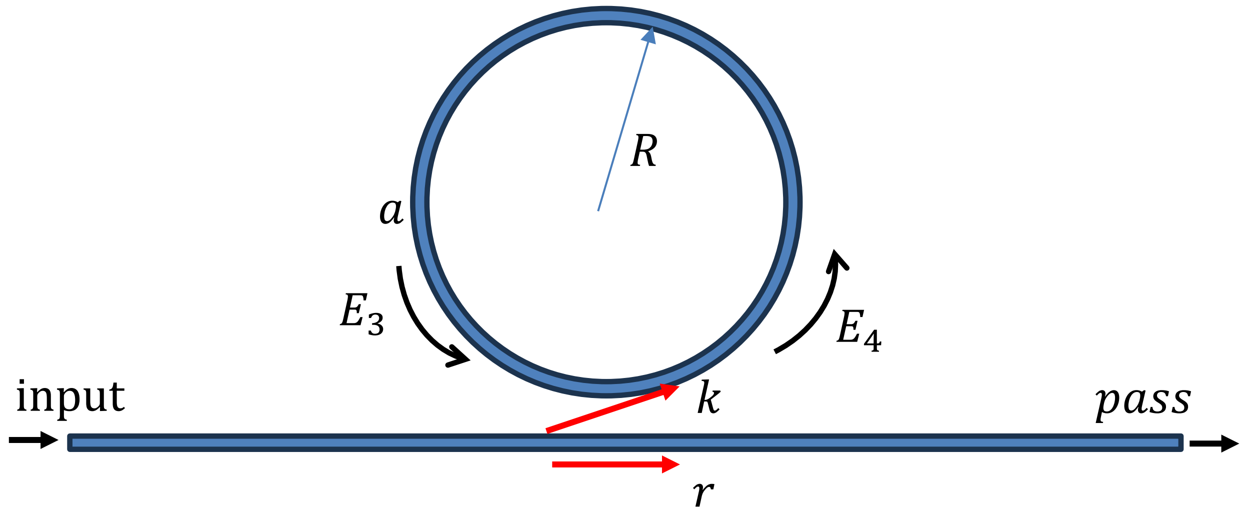
$$n_{eff}\frac{\omega_{res}}{c_0}2\pi R = 2m\pi$$

$$\omega_{res} = m\frac{c_0}{n_{eff}R}$$

$$\lambda_{res} = \frac{n_{eff}L}{m}$$

What can we do with that ? How do we get light in and out of this ring ?

# All pass filter configuration – power coupling formalism




We assume reflections are negligible:

$$\begin{bmatrix} E_4 \\ E_{pass} \end{bmatrix} = \begin{bmatrix} r & ik \\ ik & r \end{bmatrix} \begin{bmatrix} E_3 \\ E_{in} \end{bmatrix}$$

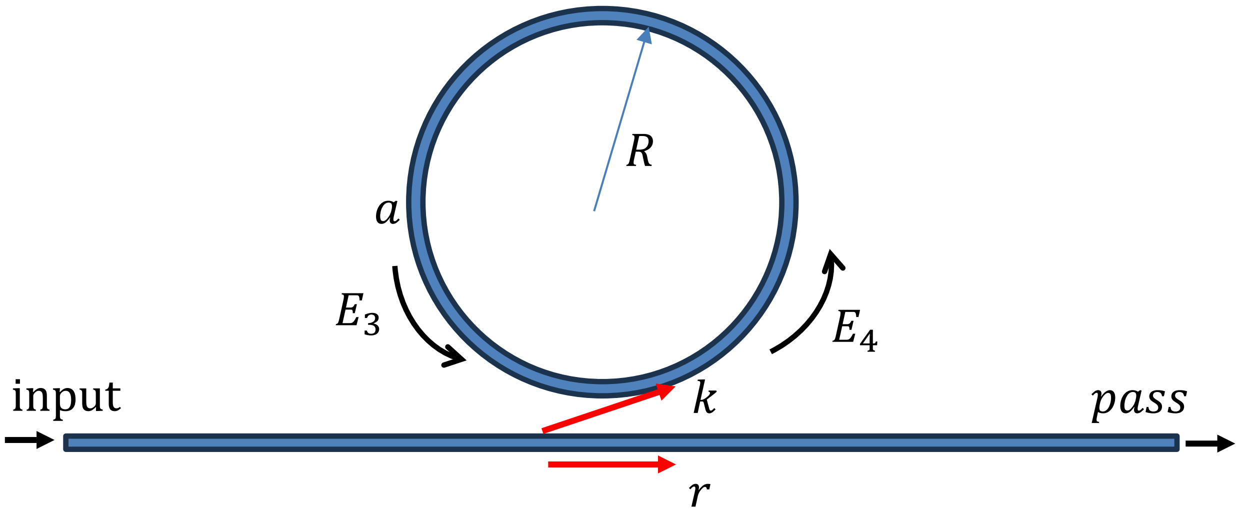
$$E_3 = ae^{i\phi}E_4$$

- $\phi = \beta L$ : the single pass phase shift
- $a$ : the single pass amplitude transmission.

  It relates to the *power attenuation coefficient* $\alpha$ (1/cm): $a^2 = \exp(-\alpha L_R)$
- $r, k$: the field self-coupling/ cross-coupling coefficient

  $r^2, k^2$ are the *power splitting ratios*, assumed to satisfy: $r^2 + k^2 = 1$

# All pass filter configuration – intensity buildup




$$\begin{bmatrix} E_4 \\ E_{pass} \end{bmatrix} = \begin{bmatrix} r & ik \\ ik & r \end{bmatrix} \begin{bmatrix} E_3 \\ E_{in} \end{bmatrix}$$

$$E_3 = ae^{i\phi}E_4$$

We can express the relationship between the intracavity intensity $E_3$ compared to the input one $E_{in}$

$$\frac{E_3}{E_{in}} = \frac{ikae^{i\phi}}{1 - rae^{i\phi}}$$

- Constructive interference at the coupler port ensures that circulating **optical intensity is built up** to a higher value that initially injected:

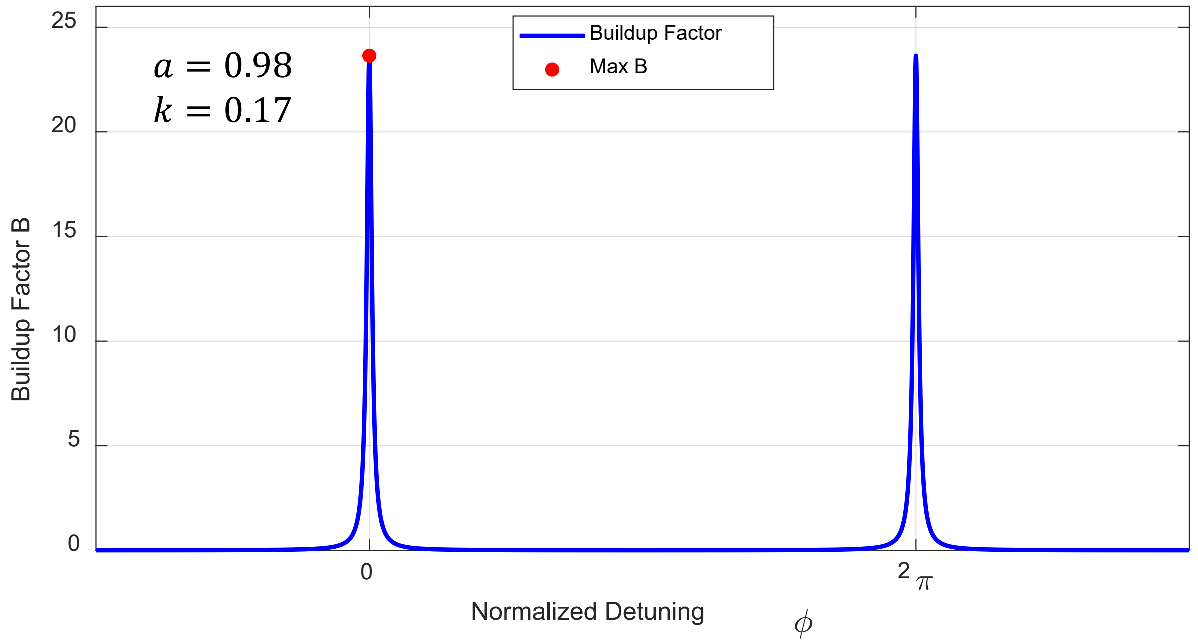
# All pass filter configuration – intensity buildup

The buildup factor $\mathcal{B}$ is given by :

$$\mathcal{B} = \frac{I_3}{I_{in}} = \left|\frac{E_3}{E_{in}}\right|^2 = \frac{(1-r^2)a^2}{1-2ra\cos\phi + r^2a^2}$$

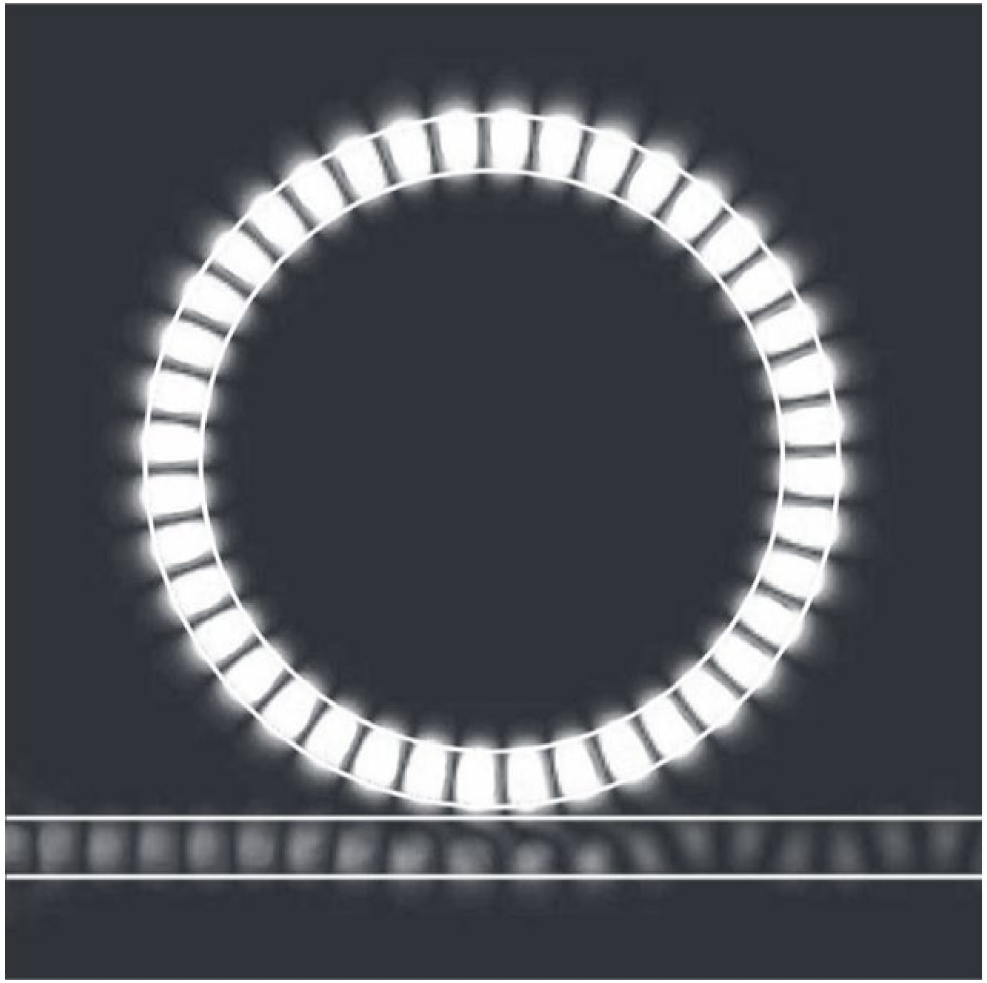
$$\xrightarrow{\phi=2m\pi} \mathcal{B} = \frac{I_3}{I_{in}} = \left|\frac{E_3}{E_{in}}\right|^2 = \frac{(1-r^2)a^2}{(1-ra)^2}$$

$$\xrightarrow{\phi=2m\pi} \mathcal{B} = \frac{k^2a^2}{(1-ra)^2}$$



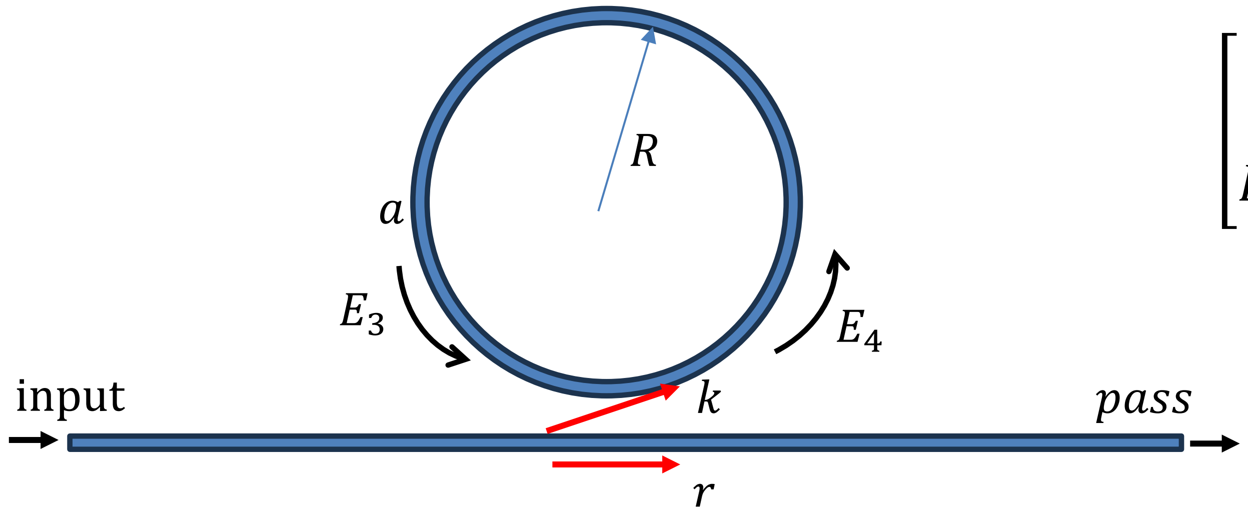
# Intensity buildup

Finite-difference time-domain simulation: intensity coherent buildup in integrated ring with index 2.5



Ref: J. Heebner, R. Grover, and T. Ibrahim, Optical Microresonators: Theory, Fabrication and Applications, Springer Series in Optical Sciences (Springer, London, 2008)

# All pass filter configuration – intensity transmission




$$\begin{bmatrix} E_4 \\ E_{pass} \end{bmatrix} = \begin{bmatrix} r & ik \\ ik & r \end{bmatrix} \begin{bmatrix} E_3 \\ E_{in} \end{bmatrix}$$

$$E_3 = ae^{i\phi} E_4$$

$$\Rightarrow \frac{E_{pass}}{E_{in}} = \frac{r - ae^{i\phi}}{1 - rae^{i\phi}}$$

We obtain the intensity transmission $T(\phi)$: $T(\phi) = \frac{I_{pass}}{I_{in}} = \frac{a^2 - 2ra\cos\phi + r^2}{1 - 2ra\cos\phi + (ra)^2}$

- $T$ is miminal at resonance, i.e. $\phi = \beta L = 2m\pi$: $T_{min} = \frac{(r-a)^2}{(1-ar)^2}$

- $T$ is maximal when $\phi = (2m+1)\pi$: $T_{max} = \frac{(a+r)^2}{(1+ar)^2}$

# Intensity transmission

If $a = r$ , $T = 0$ : critical coupling

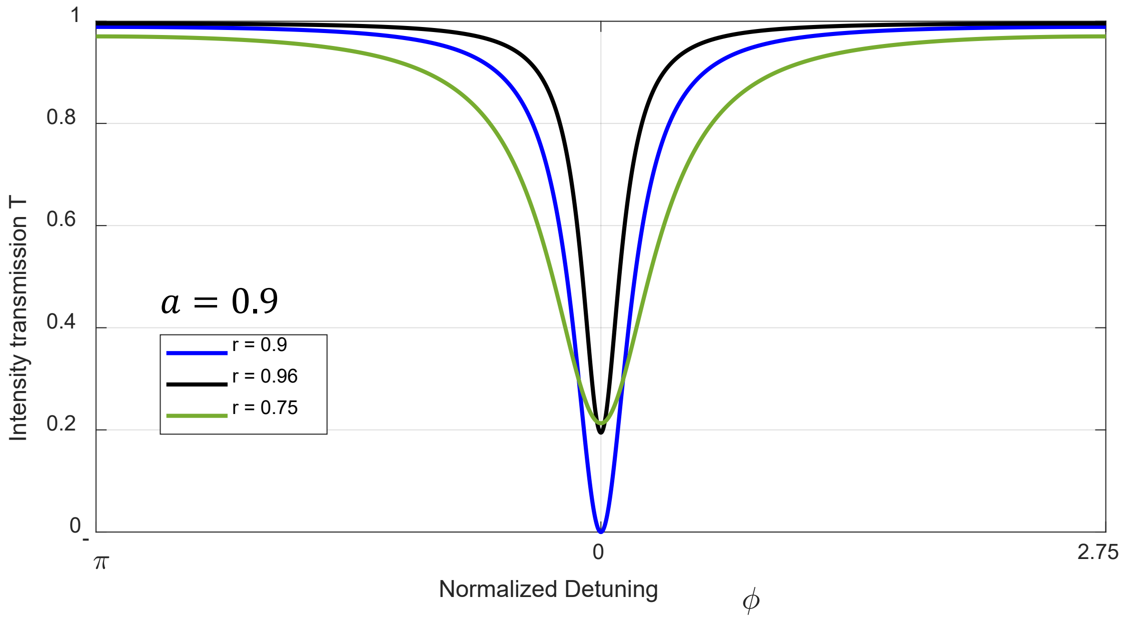
- case when coupled power is equal to the power loss in the ring ($k^2 = 1 - a^2$)

If $r < a$: overcoupling

- Case when coupled power is higher than power loss in the ring ($k^2 > 1 - a^2$)

If $r > a$: undercoupling

- Case when coupled power is lower than power loss in the ring ($k^2 < 1 - a^2$)



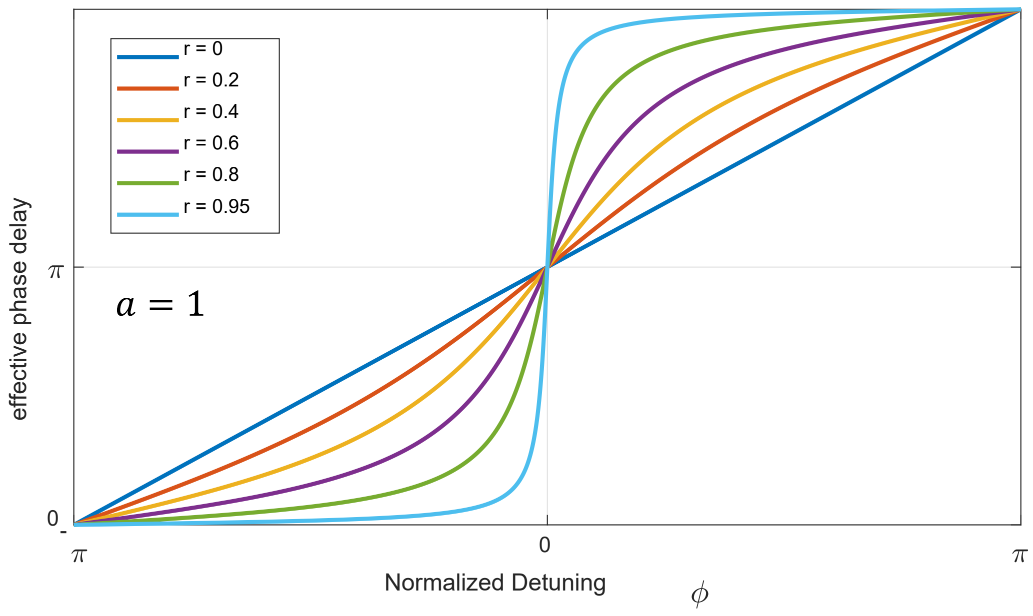
# All pass filter configuration

If $a = 1$ then the transmission is always unity …

What is it good for ? Let's consider also the phase: $\frac{E_{pass}}{E_{in}} = \frac{r - ae^{i\phi}}{1 - rae^{i\phi}} = e^{i(\pi+\phi)} \frac{a - re^{-i\phi}}{1 - rae^{i\phi}}$

$$\arg\left(\frac{E_{pass}}{E_{in}}\right) = (\pi + \phi) + \tan^{-1}\left[\frac{r \sin\phi}{a - r\cos\phi}\right] + \tan^{-1}\left[\frac{ra \sin\phi}{a - ra\cos\phi}\right]$$

$$\xrightarrow{a=1} \arg\left(\frac{E_{pass}}{E_{in}}\right) = (\pi + \phi) + 2\tan^{-1}\left[\frac{r \sin\phi}{1 - r\cos\phi}\right]$$



# Approximated transfer function

Consider the phase $\phi$ to be close to resonance condition, which is given by $\phi_0 = 2m\pi$

We define a relative phase $\varphi = \phi - \phi_0$. Therefore, we have:

$$\cos(\phi) \approx 1 - \frac{(\phi - \phi_0)^2}{2} = 1 - \frac{\varphi^2}{2}$$

The transfer function becomes: $T(\phi) = \dfrac{a^2 - 2ra\cos\phi + r^2}{1 - 2ra\cos\phi + (ra)^2}$

$$\Rightarrow T = \frac{a^2 - 2ra + ra\varphi^2 + r^2}{1 - 2ra + ra\varphi^2 + (ra)^2}$$

# Approximated transfer function

By manipulating the expression, you get $$T = 1 - \frac{V}{1 + \left(\frac{\varphi}{\varphi_0}\right)^2}$$

With $V$ the visibility: $$V = \frac{(1 - r^2)\,(1 - a^2)}{(1 - ra)^2} = \frac{k^2\,(1 - a^2)}{\left(1 - a\sqrt{1 - k^2}\right)^2}$$

And $\varphi_0$ the normalized phase $$\varphi_0 = \sqrt{\frac{(1 - ra)^2}{ra}} = \frac{1 - a\sqrt{1 - k^2}}{\left(a\sqrt{1 - k^2}\right)^{1/2}}$$

- The spectral characteristics depend on the **losses and the coupling coefficients** and can be extracted from the transmission formulas

# Spectral characteristics : resonance linewidth

We can express the full width at half maximum by evaluating $T$ at $\varphi = \pm\varphi_0$ since

$$T(\pm\varphi_0) = 1 - \frac{V}{2} = \frac{T_{min} + T_{max}}{2}$$

Let $\omega_1$ and $\omega_2$ the angular frequencies at which $\varphi = -\varphi_0$ and $\varphi = +\varphi_0$

Recall that $\varphi(\omega) = \phi(\omega) - \phi_0$ and that $\phi = \beta(\omega)$, then we can show that:

$$\beta(\omega_2) - \beta(\omega_1) = 2\varphi_0/L$$

From first order Taylor expansion of $\beta(\omega)$ around $\omega_0$, the resonance angular frequency, we also get:

$$\beta(\omega_2) - \beta(\omega_1) = \beta_1\delta\omega$$

with $\delta\omega = \omega_2 - \omega_1$ the FWHM in angular frequency

# Spectral characteristics : resonance linewidth

Given the two expressions :

$$\beta(\omega_2) - \beta(\omega_1) = \frac{2\varphi_0}{L} \text{ and } \beta(\omega_2) - \beta(\omega_1) = \beta_1 \delta\omega$$

We have that

$$\delta\omega = \frac{2\varphi_0}{L\beta_1} = \frac{2\varphi_0}{T_R}$$

$$\delta\omega = \frac{2}{T_R}\frac{(1-ra)}{\sqrt{ra}}$$

- Since $\beta_1 = \frac{1}{v_g} = \frac{n_g}{c_0}$, the inverse of the group velocity, $L\beta_1$ is the roundtrip time $T_R$
- We can also express the linewidth in wavelength :

$$\delta\lambda = \frac{\lambda^2}{2\pi c_0}\delta\omega = \frac{(1-ra)\lambda^2}{\pi c_0 T_R \sqrt{ra}} = \frac{(1-ra)\lambda^2}{\pi n_g L \sqrt{ra}}$$

# Spectral characteristics: free spectral range (FSR)

Between two resonant frequencies $\Delta\omega = \omega_{m+1} - \omega_m$, there a phase shift difference of $2\pi$ .

In other words $\beta(\omega_m)L - \beta(\omega_{m+1})L = 2\pi$

Again if we take the Taylor series around the first resonance:

$$\beta_0 - (\beta_0 + \beta_1 \Delta\omega) = \frac{2\pi}{L}$$

$$\Delta\omega = \frac{2\pi}{L\beta_1} = \frac{2\pi}{T_r} = \frac{2\pi c_0}{L n_g}$$

In wavelength the FSR is given by: $\Delta\lambda = \frac{\lambda^2}{L n_g}$

# Spectral characteristics (all pass configuration)

The on-off extinction ratio ($\Delta T$) at the output port transmission is obtained by given by :

$$\Delta T = \frac{T_{min}}{T_{max}}$$

$$T_{max} = \frac{(r+a)^2}{(1+ra)^2} \qquad T_{min} = \frac{(a-r)^2}{(1-ra)^2}$$

The Finesse $\mathfrak{I}$ is defined as the **ratio of FSR to the resonance width**:

$$\mathfrak{I} = \frac{\Delta\omega}{\delta\omega} = \frac{\pi\sqrt{ra}}{1-ra}$$

- Physical meaning: The Finesse represents within a factor $2\pi$ the number of round trips made by light in the ring before its energy is reduced by a factor $1/e$ of its initial value

# Quality factor ($Q$-factor)

The $Q$-factor is a measure of the sharpness of the resonance relative to its central frequency:

$$Q = \frac{\lambda_{res}}{\delta\lambda}$$

$$Q = \frac{\pi n_g L\sqrt{ra}}{\lambda_{res}(1-ra)} = \frac{\omega_{res}\mathfrak{I}}{\Delta\omega}$$

Physical meaning: represents the number of oscillations of the field before the circulating energy is depleted to $1/e$ of its initial energy

- Microring is excited and the rate of power decay is considered
- Loss factors must be reduced to have high $Q$ resonances

We distinguish two types of $Q$-factors:

- Loaded $Q$-factor: coupled resonator (intrinsic and coupling losses) ($Q$, sometimes written $Q_L$)
- Intrinsic $Q$-factor : when resonator would not be coupled to a waveguide (only intrinsic losses) ($Q_i$)

# Illustration

$$\delta\omega = \frac{2}{T_R}\frac{(1-ra)}{\sqrt{ra}} \qquad Q = \frac{\pi n_g L\sqrt{ra}}{\lambda_{res}(1-ra)} \qquad \mathfrak{I} = \frac{\pi\sqrt{ra}}{1-ra}$$

FSR: 50 GHz
Finesse: 157
$Q = 4.6 \cdot 10^3$

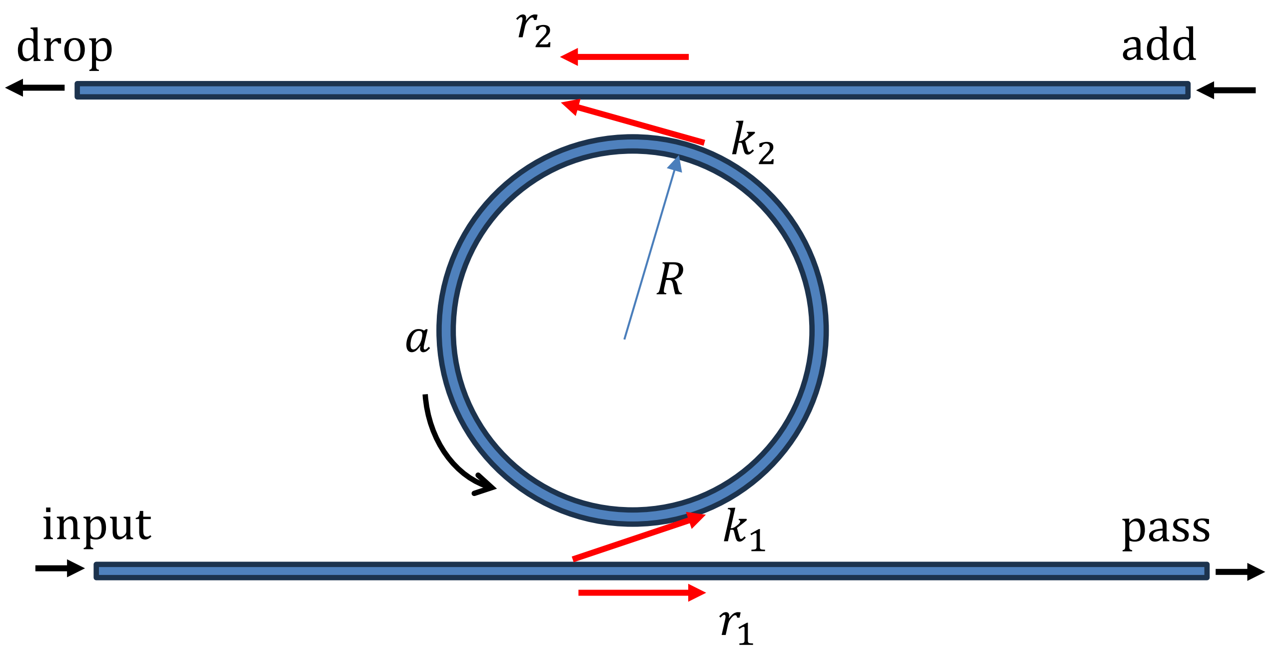
FSR: 10 GHz
Finesse: 157
$\boldsymbol{Q = 2.4 \cdot 10^4}$

FSR: 50 GHz
**Finesse: 773**
$Q = 2.4 \cdot 10^4$

Keep same intrinsic and coupling rates
Change only the FSR

Keep the FSR
Change only the intrinsic and coupling rates

# Add-drop configuration




Ring resonator is coupled to 2 waveguides

- Incident field is partly transmitted to the drop port

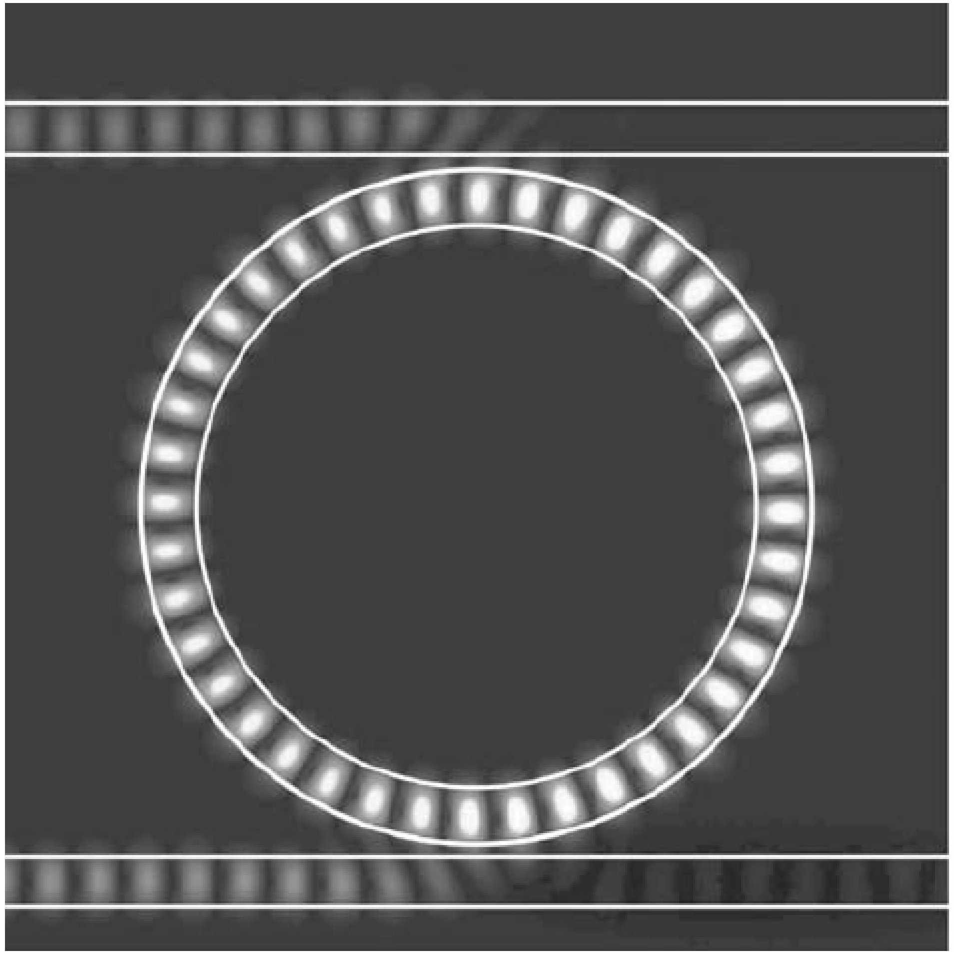
Can derive the transmission to pass and drop ports from CW operation

$$T_p = \frac{I_{pass}}{I_{in}} = \frac{r_2^2 a^2 - 2r_1 r_2 a \cos\beta L + r_1^2}{1 - 2r_1 r_2 a \cos\beta L + (r_1 r_2 a)^2}$$

$$T_d = \frac{I_{drop}}{I_{in}} = \frac{(1 - r_1^2)(1 - r_2^2)a}{1 - 2r_1 r_2 a \cos\beta L + (r_1 r_2 a)^2}$$

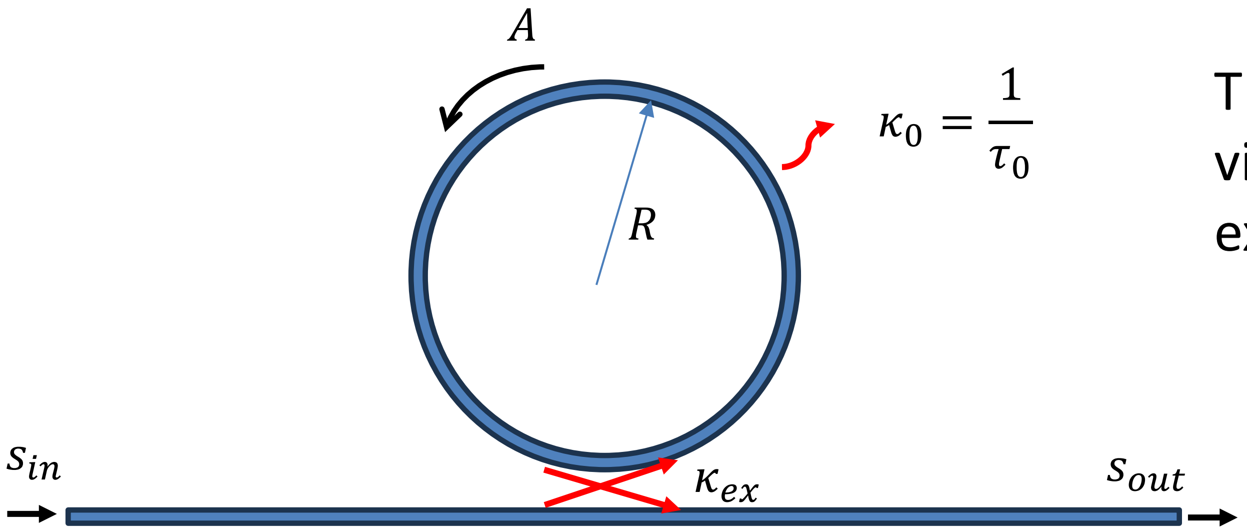
# Intensity build up and rerouting

Finite-difference time-domain simulation: intensity coherent buildup in integrated ring with index 2.5



# Principle of microring resonators – Temporal coupled mode theory

# Temporal coupled mode theory approach




TMCT describes device behavior from the point of view of **energy exchange** between resonator and external waveguide

- Only valid under assumption of weak coupling
- Spatial distribution of energy in the cavity is considered uniform

$A(t)$ describes the amplitude of the field inside a mode of the resonator, $|A|^2$is the energy stored

$|s_{in}|^2$ represents the photon flux of the pump and relates to the incident power.

In the simplest picture, there are only two rates involved:

- The coupling rate $\kappa_{ex}$ gives the strength of the coupling between the mode outside/inside the resonator
- The intrinsic loss rate $\kappa_0$, the combined loss rate inside the resonator to all relevant loss channels

# Field evolution without driving/coupling

When only considering intrinsic dynamics (no coupling) of the mode in its rotating frame then:

$$\frac{dA(t)}{dt} = -\frac{\kappa_0}{2}\mathrm{A(t)}$$

Solution is an exponential decaying field, such that the photon number decays as:

$$|A(t)|^2 = \exp(-\kappa_0 t)$$

Since the loss in one round trip is $(1 - \exp(-\alpha L_R))$, then the decay rate $\kappa_0$ is:

$$\kappa_0 = \frac{1 - \exp(-\alpha L_R)}{T_R} \approx \frac{\alpha L_R}{T_R}$$

# Field evolution with driving

Need to take into account the phase

- Assume a cavity mode $A(t)\exp(-i\omega_m t)$, $\omega_m$ angular frequency of a resonance
- Assume a driving field $s_{in}\exp\left(-i\omega_p t\right)$ , $\omega_p$ angular frequency of the pump light

Equation of motion for the complex amplitude of the cavity field in the rotation frame of $\omega_m$:

$$\frac{dA(t)}{dt} = -\frac{(\kappa_0 + \kappa_{ex})}{2}A(t) + \sqrt{\kappa_{ex}}s_{in}e^{-i(\omega_p - \omega_m)t}$$

Change to the rotating frame to the driving field and introduce total energy loss $\kappa = \kappa_0 + \kappa_{ex}$

$$\frac{dA(t)}{dt} = -\left[-i\left(\omega_m - \omega_p\right) - \frac{\kappa}{2}\right]A(t) + \sqrt{\kappa_{ex}}s_{in}$$

Note, we call coupling ratio: $\eta = \kappa_{ex}/\kappa$

# Field evolution with driving

In the steady stade, we can derive the cavity field amplitude:

$$A = \frac{\sqrt{\kappa_{ex}}}{i(\omega_m - \omega_p) + \frac{\kappa}{2}} s_{in}$$

The photon number inside the vacity is therefore :

$$|A|^2 = \frac{\kappa_{ex}}{(\omega_m - \omega_p)^2 + \left(\frac{\kappa}{2}\right)^2} |s_{in}|^2$$

Energy conservation dictates that $|s_{out}|^2 = |s_{in}|^2 - \kappa_0 |A|^2$, we get

$$T(\omega) = \left|\frac{s_{out}}{s_{in}}\right|^2 = 1 - \frac{\kappa_{ex}\kappa_0}{(\omega_m - \omega)^2 + \left(\frac{\kappa}{2}\right)^2}$$

# Transmission and characteristics

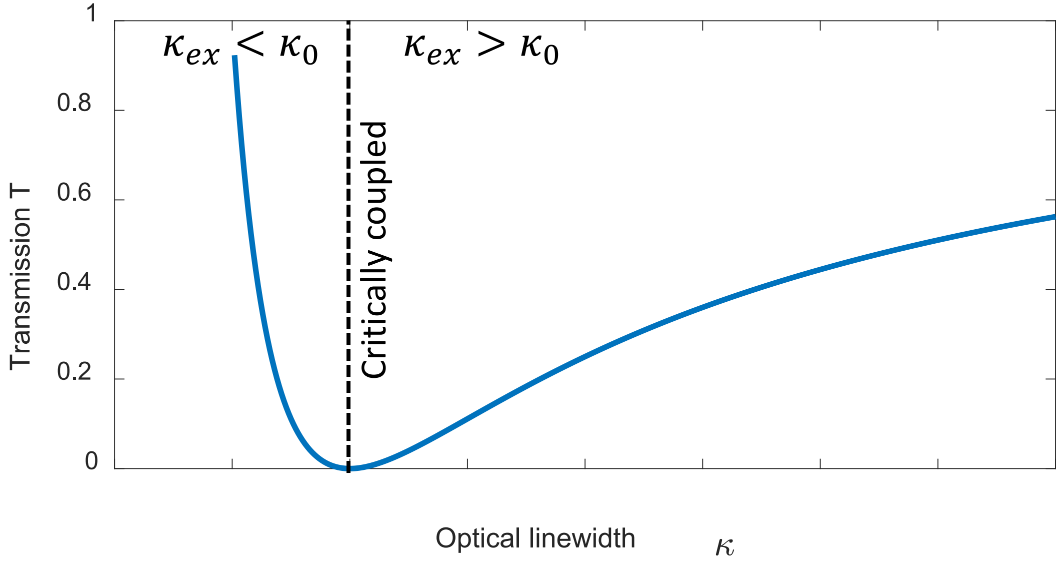
$$\left|\frac{s_{out}}{s_{in}}\right|^2 = 1 - \frac{\kappa_{ex}\kappa_0}{\left(\frac{\kappa}{2}\right)^2 + (\omega - \omega_{res})^2}$$

Lorentzian dip with minimal transmission on resonance ($\omega - \omega_{res} = 0$)

- FWHM (linewidth of the resonance): $\delta\omega = \kappa$ (and $\delta\nu = \kappa/2\pi$ in frequency)
- Transmission extinction ratio: $\Delta T = \left(\frac{\kappa_0+\kappa_{ex}}{\kappa_0-\kappa_{ex}}\right)^2$

Coupling conditions

- Undercoupling $\eta < \frac{1}{2}$ $(\kappa_{ex} < \kappa_0)$
- Critical coupling $\eta = \frac{1}{2}$ $(\kappa_{ex} = \kappa_0)$
- Overcoupling $\eta > \frac{1}{2}$ $(\kappa_{ex} > \kappa_0)$



# Quality factor

We know that : $Q = \frac{\lambda_{res}}{\delta\lambda}$

In angular frequency we have $Q = \frac{\omega_{res}}{\delta\omega} = \frac{\omega_{res}}{\kappa}$

Give than the total loss rate includes the intrinsic and coupling rates ($\kappa = \kappa_0 + \kappa_{ex}$), we can express the intrinsic $Q$ factor ($Q_i$) and coupling $Q$ factor ($Q_c$)

$$Q_i = \frac{\omega_{res}}{\kappa_0}$$

$$Q_c = \frac{\omega_{res}}{\kappa_{ex}}$$

$$Q = \left(\frac{1}{Q_c} + \frac{1}{Q_i}\right)^{-1}$$

# Finesse and $Q$ factor

By definition we have seen that $\mathfrak{J} = \frac{\Delta\omega}{\delta\omega}$

We therefore have a link with the $Q$ factor which is:

$$Q = \frac{\mathfrak{J}}{\Delta\omega}\omega_{res}$$

# Link to experimental characterization

The measure of the transmission of a resonator can be used to extract important parameters :

$$Q = \frac{\omega_{res}}{\delta\omega}$$

$$Q_i = \frac{2Q}{1 \pm \sqrt{T_{min}/T_{max}}}$$

($+$ in case of undercoupled and $-$ in case of overcoupled)

We can then estimate the intrinsic loss of the resonator using:

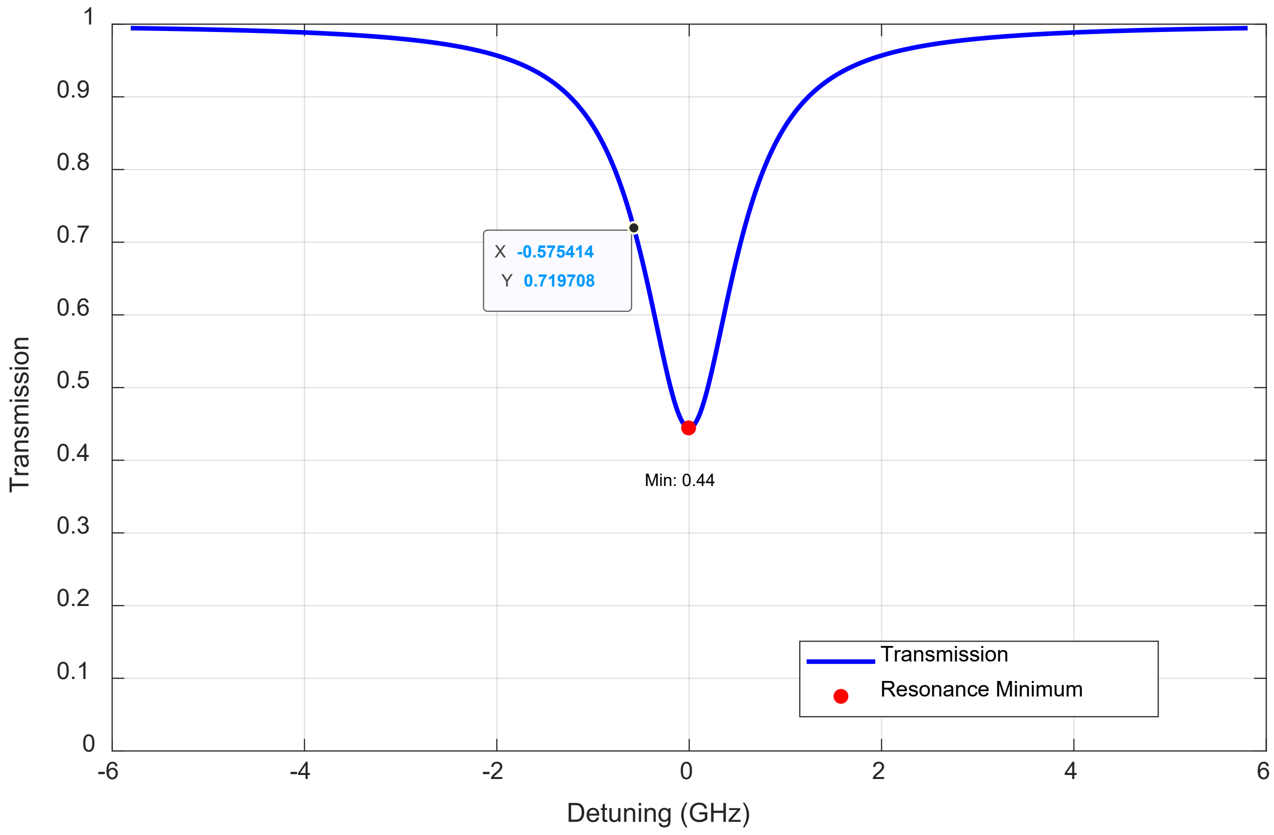
$$\alpha = \frac{\omega_{res}}{Q_i v_g}$$

# Example

Wavelength : 1550 nm

$n_{eff}$ = 1.5

Overcoupled



# Microring resonator modulators

# Operating principle

In a ring resonator, modulation can be obtained through a resonance shift caused by a change of refractive index:

$$\Delta\omega_{res} = \frac{\Delta n_{eff}}{n_g}\omega_{res}$$

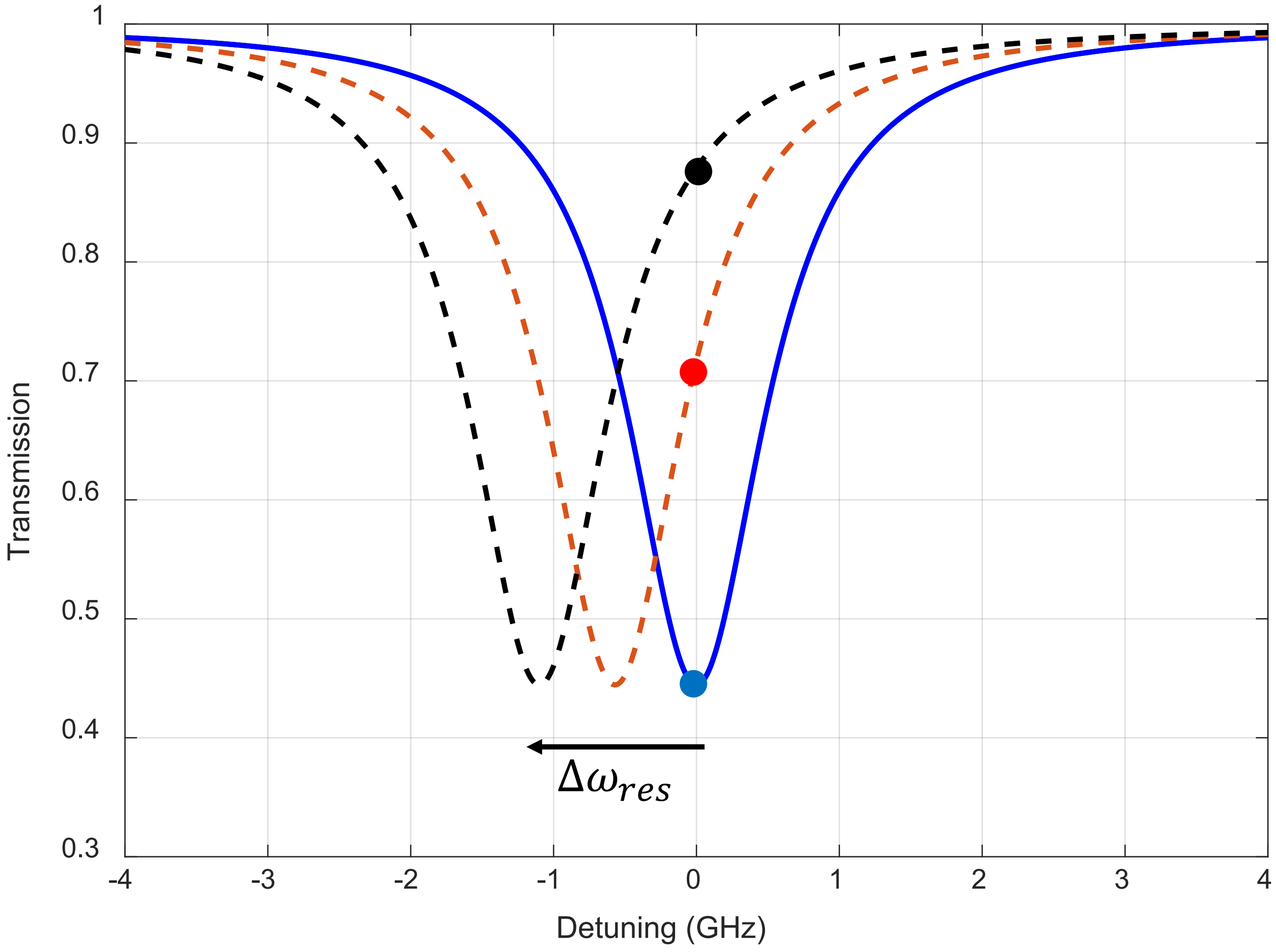
- Inclusion of group index $n_g$ : effective index changes both due to refractive index modification but also due to the resonant wavelength change

Recall that the FWHM of a resonance is a function of the quality factor

$$\text{FWHM} = \frac{\omega_{res}}{Q}$$

- Gives the characteristic quantity by which the resonance needs to be shifted to obtain substantial modulation

# Resonance shift




The higher the $Q$-factor, the lower the resonance shift required

- i.e. the lower $\Delta n_{eff}$ needed
- Devices with high $Q$ can be very efficient

But ....

Optically narrow device:

- Resonant enhancement constrains carrier to coincide with resonant frequency

Low speed device:

- Field in resonators decays with a $1/e$ time constant given by $2Q/\omega_{res}$

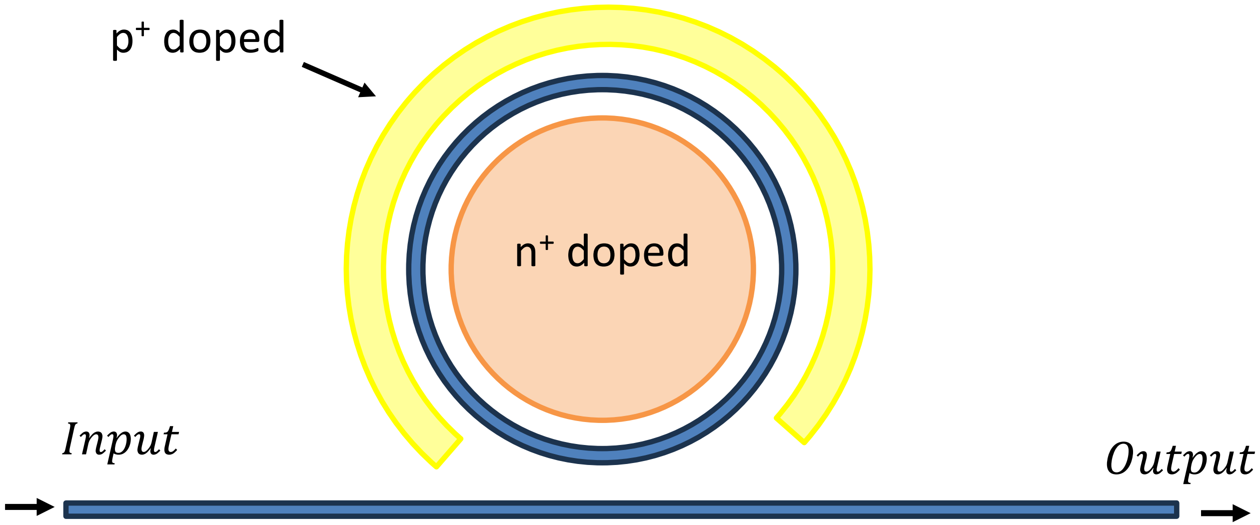
# Speed consideration

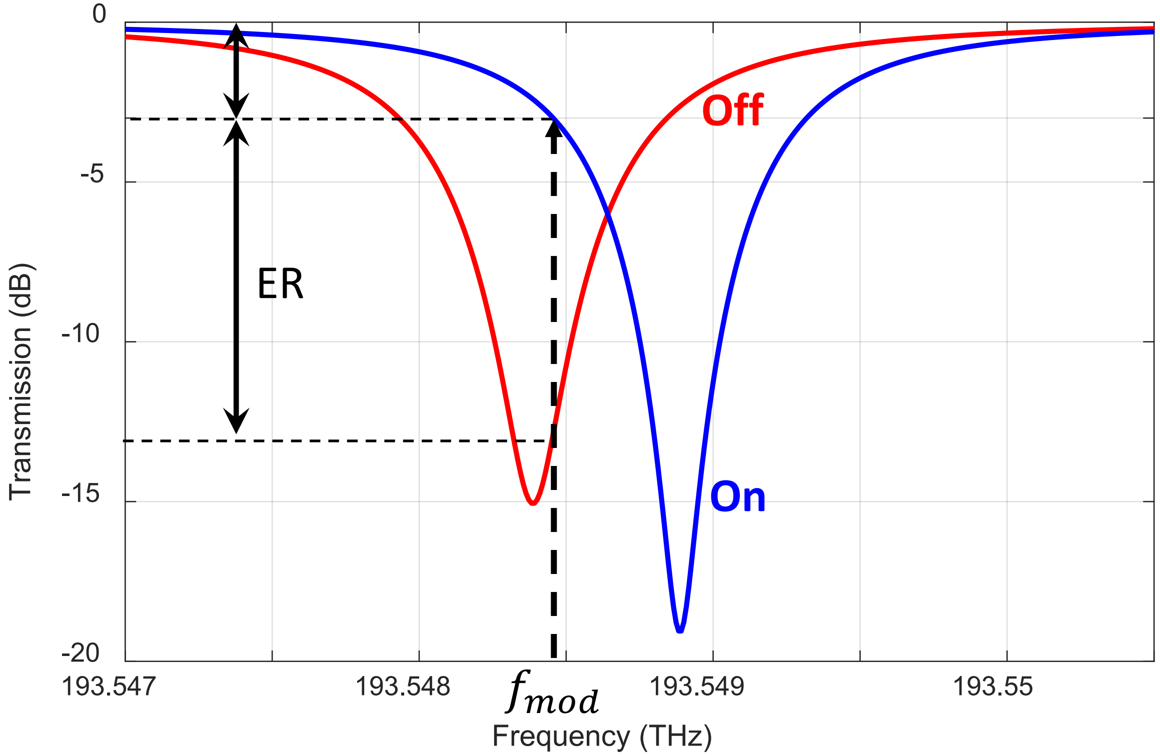
For highspeed devices: it is necessary to reduce the $Q$-factor to obtain low enough photon storage time, thus reducing the modulation efficiency

$$\tau_p = \frac{Q}{\omega_{res}} = \frac{\lambda Q}{2\pi c}$$

$$\Delta f = \frac{1}{\sqrt{\left(2\pi\tau_p\right)^2 + (2\pi RC)^2}}$$

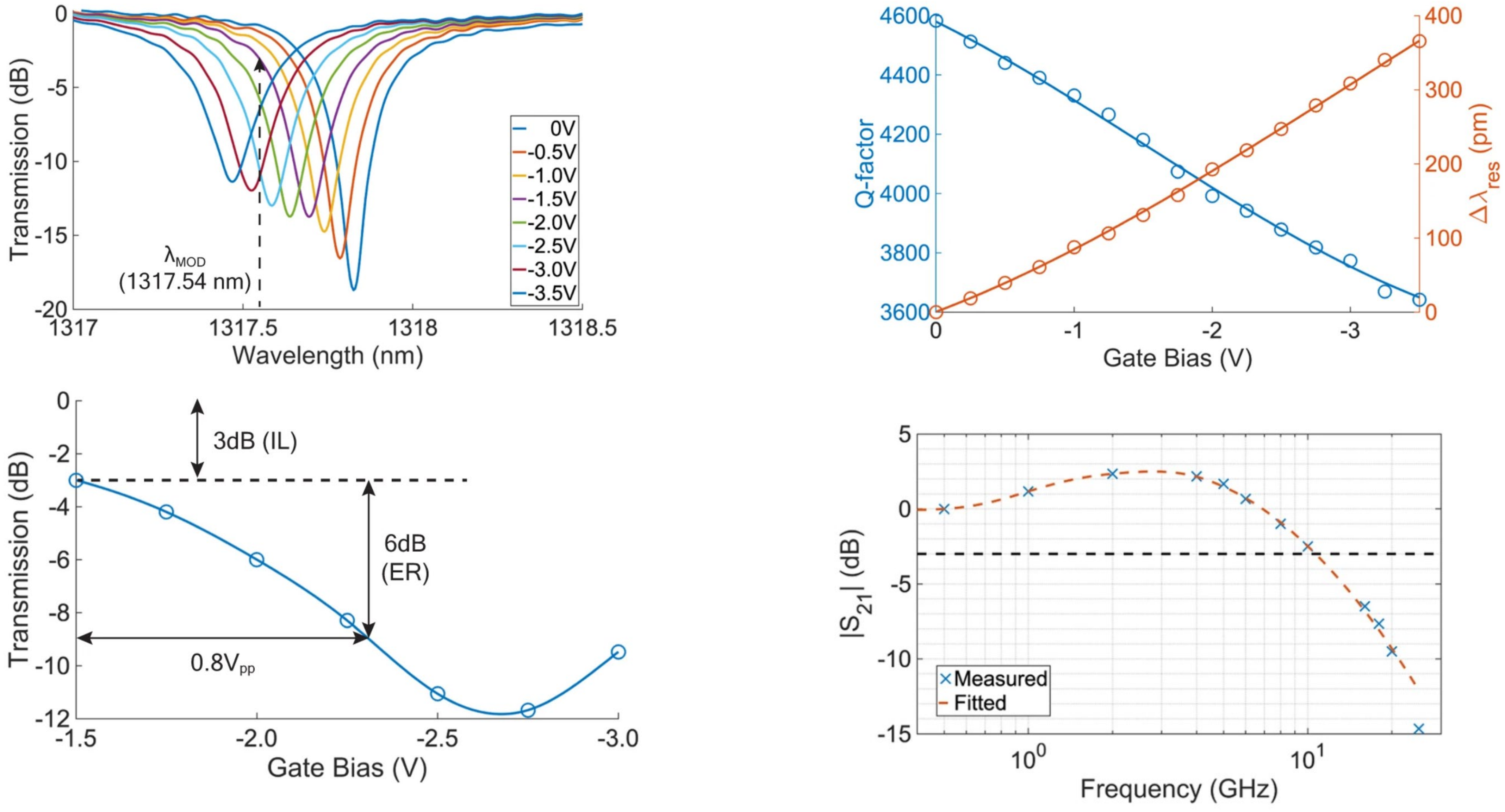
# Operation


p+ doped
n+ doped
Input
Output


Insertion loss
ER
Off
On
Transmission (dB)
0
-5
-10
-15
-20
193.547
193.548
$f_{mod}$
193.549
193.55
Frequency (THz)


Off
On

# Example from scientific article




Wei-Che Hsu et al., Nature Communications 15(826), 2024